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# DEFORMATION OF POROELASTIC HALF-SPACE DUE TO TENSILE DISLOCATION

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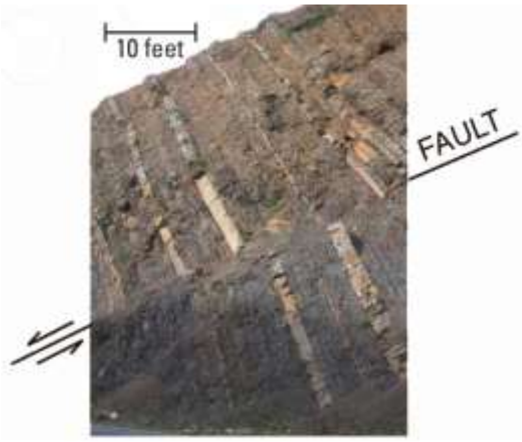
## ABSTRACT
The closed-form analytical expressions for the displacements and strains due to Tensile Dislocation (TD) located in a homogeneous, isotropic poroelastic half-space are obtained. The variation of radial displacement and strains with epicentral distance for the various materials Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone and Westerly Granite are discussed.

**KEYWORDS**: Poroelastic half-space, displacement, epicentral distance, Tensile Dislocation

## I. INTRODUCTION
A fault can be defined as the movement of the rock on either side. When that movement is sudden, the released energy causes an earthquake. These fault systems are the boundaries of the huge plates that make up the Earth's crust. A fault plane is the plane formed between the two rock blocks that slip one with the other during an earthquake. This can occur in any direction with the blocks moving away from each other. Faults occur from both tensional and compressional forces.

**Figure:**




***a small fault in San Mateo County.***

***Matching layers across the fault shows that it has offset the***

***Sandstone (lighter layers) and shale (darker layers) about ten feet.***

A successful model of earthquake-related deformation might be described as one that is straightforward, versatile, and computationally efficient. Model solutions should optimally describe deformation in three dimensions to allow for geometric complexities associated with planar (horizontal) and vertical (depth) displacements. Tensile fault representation has several very important geophysical applications, such as modelling of the deformation field due to a dyke injection in the volcanic region, mine collapse and fluid driven crack.

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Since tensile cracks in a heterogeneous media have received much less attention than shear crack models and tensile cracks are important in geophysical modelling because they are reasonable mathematical models for magma ascent in rift zones or volcanic areas. Moreover, the study of the displacement field due to a rectangular tension crack have large number of applications in the field of mechanical engineering Therefore, using the procedure adopted by Kumari *et al.* (1992), we derive the displacement field due to a point tensile dislocation source placed in a two phase medium consisting of two homogeneous, isotropic, perfectly elastic half-spaces in welded contact. Integrating these results over the fault area, we derive the closed form expressions for the displacements caused by a finite rectangular tensile fault placed parallel to the interface between two uniform half-spaces in welded contact.

## II. THEORY

A dislocation is defined as a surface S in the medium across which there is discontinuity $\Delta u_j$ in the displacement field of the type $\Delta u_j = u_j^+ - u_j^- = U_0 e_j$, where $u_j^+$ and $u_j^-$ are the displacements of the two faces of the cut, $U_0$ is the magnitude of dislocation and ***e*** is a unit vector in the direction of the dislocation. Steketee (1958a) showed that the displacement field $u_i(\mathbf{x})$ due to a dislocation $\Delta u_j(\mathbf{y}) = U_0 e_j$ across a surface S in an isotropic medium is given by (Singh et al. 2000)

$$u_i = \int_S \Delta u_j \left[ \lambda \delta_{jk} \frac{\partial u_i^m}{\partial y_m} + \mu \left( \frac{\partial u_i^j}{\partial y_k} + \frac{\partial u_i^k}{\partial y_j} \right) \right] n_k \, dS \qquad (1)$$

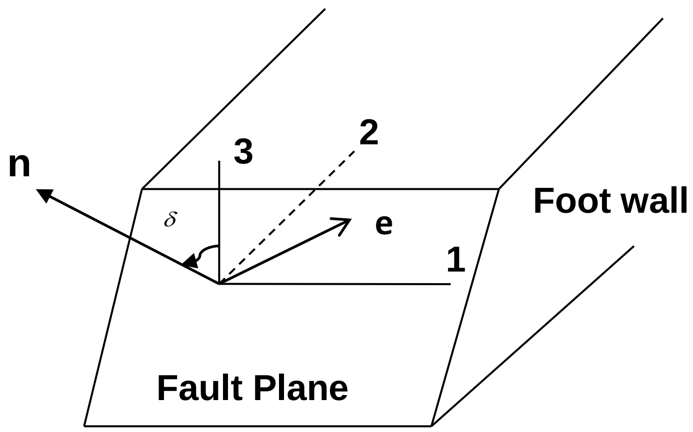
where ( **y** ) is a point on S , $n_k$ are the direction cosines of the normal to the surface element dS and $u_i^j$ is the $x_i$- component of the displacement at (**x**) due to a concentrated force of unit magnitude acting at the point (**y**) of the medium in the $x_j$- direction. For a point dislocation source, equation (1) becomes

$$u_i = \lambda U_0 e_j n_j \left[ \frac{\partial u_i^m}{\partial y_m} \right] dS + \mu U_0 e_j n_k \left( \frac{\partial u_i^j}{\partial y_k} + \frac{\partial u_i^k}{\partial y_j} \right) dS \qquad (2)$$

where *dS* is the area of the point dislocation. Let the $x_1$-axis be taken along the strike of the fault and $x_3$-axis vertically upward and δ is the dip angle (Figure 1). For a point tensile dislocation **e** = **n** and δ=0, hence, from equation (2), the displacement field for a point tensile dislocation is expressed as

$$u_{(n)i} = U_0 \left[ \lambda \left( \frac{\partial u_i^1}{\partial y_1} + \frac{\partial u_i^2}{\partial y_2} \right) + (\lambda + 2\mu) \left( \frac{\partial u_i^3}{\partial y_3} \right) \right] dS$$

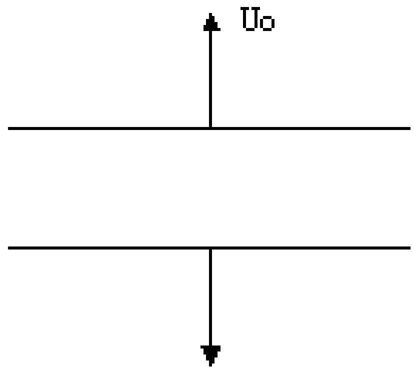
**Figure 1:**

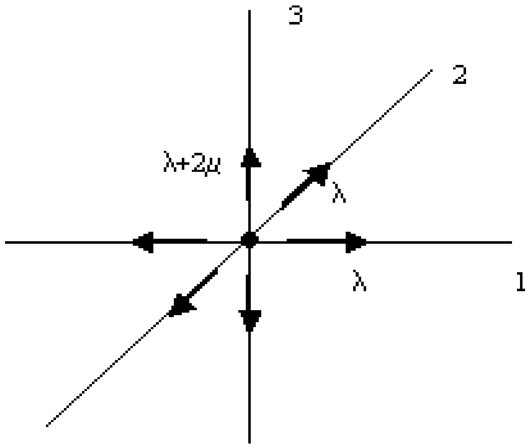



***Geometry of a point shear dislocation, n is a unit vector normal to the fault plane and gives the direction of motion of the hanging wall relative to the foot wall***

Therefore, a tensile dislocation $U_0$ (Figure 2) on a horizontal planer element of area dS is equivalent to a vertical dipole of moment $(\lambda+2\mu)U_0dS$ plus two mutually orthogonal horizontal dipoles of moment $\lambda U_0dS$ each (Figure 3) (Ben-Menahem and Singh, 1981).

**Figure 2:** **Figure 3:**

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*DISPLACEMENTS*

Using displacement field given by Amit Kumar *et al.* (2012), we obtained the expressions for the displacement components due to Tensile Dislocation of magnitude **P** acting at the point (0, 0, c) in poroelastic half-space.

$$u_i = U_0\left[\lambda\left(\frac{\partial u_i^1}{\partial x_1}+\frac{\partial u_i^2}{\partial x_2}\right)+(\lambda+2\mu)\left(\frac{\partial u_i^3}{\partial x_3}\right)\right]dS \quad (4)$$

$where \; i = 1,2,3$

$$u_1 = \frac{PU_0 ds\, x_1}{8\pi}\left[\begin{array}{l}\frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-1}{R_1^3}+\frac{1}{R_2^3}\right)\hat{\sigma}+\frac{12c(x_3+c)}{R_2^5}\\ +6cx_3\bar{\sigma}\left(\frac{1}{R_2^5}-\frac{5(x_3+c)^2}{R_2^7}\right)\end{array}\right\}\\ +\bar{\sigma}\left\{\begin{array}{l}\frac{1}{R_1^3}-\frac{3(x_3-c)^2}{R_1^5}\\ +\frac{\left(8(1-\tilde{\sigma})^2-1\right)}{R_2^3}\\ +\frac{3\{-(3-4\tilde{\sigma})(x_3+c)(x_3-c)+2cx_3+2c(x_3+c)\}}{R_2^5}\\ -\frac{30cx_3(x_3+c)^2}{R_2^7}\end{array}\right\}\end{array}\right] \quad (5)$$

$u_2$ can be obtained by replacing $x_1$ to $x_2$ in eqn. (5)

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$$u_3 = \frac{PU_0 ds}{8\pi}\left[\begin{array}{l}\frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-(x_3-c)}{R_1^3}+\frac{(x_3+c)}{R_2^3}-\frac{6c(x_3+c)^2}{R_2^5}\right)\hat{\sigma}\\ +\bar{\sigma}6cx_3(x_3+c)\left(\frac{3}{R_2^5}-\frac{5(x_3+c)^2}{R_2^7}\right)\end{array}\right\}\\ +\bar{\sigma}\left\{\begin{array}{l}\frac{-(1-4\tilde{\sigma})(x_3-c)}{R_1^3}-\frac{3(x_3-c)^3}{R_1^5}\\ +\frac{4(1-2\tilde{\sigma})(\tilde{\sigma}(x_3+c)-c)+(x_3-c)}{R_2^3}\\ \frac{3(x_3+c)(6cx_3+2c(x_3+c))}{R_2^5}\\ -3(x_3+c)^3\left(\frac{(3-4\tilde{\sigma})}{R_2^5}+\frac{10cx_3}{R_2^7}\right)\end{array}\right\}\end{array}\right] \quad (6)$$

Where $R_1^2 = x_1^2 + x_2^2 + (x_3 - c)^2$ ; $R_2^2 = x_1^2 + x_2^2 + (x_3 + c)^2$ (7)

$\kappa = \lambda + \frac{2}{3}\mu$ is bulk-modulus; $\lambda$ , $\mu$ are Lamé constants;

$\hat{\sigma} = \frac{1-2\tilde{\sigma}}{1-\tilde{\sigma}}$; $\bar{\sigma} = \frac{1}{1-\tilde{\sigma}}$; $\tilde{\sigma} = \frac{\tilde{\lambda}}{2(\tilde{\lambda}+\mu)}$ is Poisson ratio. If the porosity disappears, then $\tilde{\lambda} \to \lambda$

*Case 1: when z ≠ 0*

Using $x_1 = r\cos\theta$; $x_2 = r\sin\theta$; $x_3 = z$ in eqn (7) we get

$R_1^2 = r^2 + (z-c)^2$, $R_2^2 = r^2 + (z+c)^2$

The displacements components in cylindrical co-ordinates are given as

$u_r = u_1\cos\theta + u_2\sin\theta$ ; $u_\theta = u_1\sin\theta - u_2\cos\theta$ ; $u_z = u_3$

$$u_r = \frac{PU_0 ds\, r}{8\pi}\left[\begin{array}{l}\frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-1}{R_1^3}+\frac{1}{R_2^3}\right)\hat{\sigma}+\frac{12c(z+c)}{R_2^5}\\ +6cz\bar{\sigma}\left(\frac{1}{R_2^5}-\frac{5(z+c)^2}{R_2^7}\right)\end{array}\right\}\\ +\bar{\sigma}\left\{\begin{array}{l}\frac{1}{R_1^3}-\frac{3(z-c)^2}{R_1^5}\\ +\frac{(8(1-\tilde{\sigma})^2-1)}{R_2^3}\\ +\frac{3\{-(3-4\tilde{\sigma})(z+c)(z-c)+2cz+2c(z+c)\}}{R_2^5}\\ -\frac{30cz(z+c)^2}{R_2^7}\end{array}\right\}\end{array}\right] \quad (8)$$

$$u_\theta = 0 \quad (9)$$

$$u_z = \frac{PU_0 ds}{8\pi}\left[\begin{array}{l} \frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-(z-c)}{R_1^3}+\frac{(z+c)}{R_2^3}-\frac{6c(z+c)^2}{R_2^5}\right)\hat{\sigma} \\ +\bar{\sigma}6cz(z+c)\left(\frac{3}{R_2^5}-\frac{5(z+c)^2}{R_2^7}\right)\end{array}\right\} \\ +\bar{\sigma}\left\{\begin{array}{l}\frac{-(1-4\tilde{\sigma})(z-c)}{R_1^3}-\frac{3(z-c)^3}{R_1^5} \\ +\frac{4(1-2\tilde{\sigma})(\tilde{\sigma}(z+c)-c)+(z-c)}{R_2^3} \\ \frac{3(z+c)(6cz+2c(z+c))}{R_2^5} \\ -3(z+c)^3\left(\frac{(3-4\tilde{\sigma})}{R_2^5}+\frac{10cz}{R_2^7}\right)\end{array}\right\}\end{array}\right] \qquad (10)$$

*STRAINS*

Strains in cylindrical co-ordinates can be calculated using $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$

On using equations (8) - (10) in the strain displacement relations, the following expressions of strains for a point tensile dislocation in a poroelastic medium are obtained as:

$$e_{rr} = \frac{PU_0 ds}{8\pi}\left[\begin{array}{l} \frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-1}{R_1^3}+\frac{1}{R_2^3}+3r^2\left(\frac{1}{R_1^5}-\frac{1}{R_2^5}\right)\right)\hat{\sigma} \\ +12c(z+c)\left(\frac{1}{R_2^5}-\frac{5r^2}{R_2^7}\right) \\ +6cz\bar{\sigma}\left(\frac{-4}{R_2^5}+\frac{35r^2(z+c)^2}{R_2^9}\right)\end{array}\right\} \\ +\bar{\sigma}\left\{\begin{array}{l}\frac{4}{R_1^3}+\frac{15r^2(z-c)^2}{R_1^7} \\ +\left(8(1-\tilde{\sigma})^2-1\right)\left(\frac{1}{R_2^3}-\frac{r^2}{R_2^5}\right) \\ +\frac{3(z+c)(2c-8cz-3(3-4\tilde{\sigma})(z-c))}{R_2^5} \\ +15r^2(z+c)\left(\frac{(3-4\tilde{\sigma})(z-c)-2c}{R_2^7}+\frac{14cz(z+c)}{R_2^9}\right)\end{array}\right\}\end{array}\right] \qquad (11)$$

$$e_{zz}=\frac{PU_0 ds}{8\pi}\left[\begin{array}{l}\frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{-1}{R_1^3}+\frac{1}{R_2^3}+\frac{3(z-c)^2}{R_1^5}-\frac{3(z+c)^2}{R_2^5}\right)\hat{\sigma}\\ +\bar{\sigma}\left(\begin{array}{l}\frac{2c(z+c)(1+4\tilde{\sigma})+6cz}{R_2^5}\\ +30c(z+c)^2\left(\frac{-2(3z+\tilde{\sigma}(z+c))}{R_2^7}+\frac{7z(z+c)^2}{R_2^9}\right)\end{array}\right)\end{array}\right\}\\ +\bar{\sigma}\left\{\begin{array}{l}\frac{-(1-4\tilde{\sigma})}{R_1^3}+\frac{15(z-c)^4}{R_1^7}\\ -\frac{6(1+2\tilde{\sigma})(z-c)^2}{R_1^5}+\frac{(4(1-2\tilde{\sigma})\tilde{\sigma}+1)}{R_2^3}\\ +\frac{3((z+c)(8c(1-\tilde{\sigma})+6c+(z-c))+6cz+((z+c)^2(8\tilde{\sigma}(1+\tilde{\sigma})-9)))}{R_2^5}\\ +\frac{60z(z+c)^2((z+c)-3c)}{R_2^7}-15(z+c)^4\left(\frac{(1+4\tilde{\sigma})}{R_2^7}-\frac{14cz}{R_2^9}\right)\end{array}\right\}\end{array}\right] \tag{12}$$

$$e_{rz}=\frac{3\Pr U_0 ds}{8\pi}\left[\begin{array}{l}\frac{\lambda}{\mu}\left\{\begin{array}{l}\left(\frac{(z-c)}{R_1^5}-\frac{z}{R_2^5}\right)\hat{\sigma}\\ +c\bar{\sigma}\left(\begin{array}{l}\frac{2}{R_2^5}-\frac{(z+c)((1+9\tilde{\sigma})(z+c)+30z)}{R_2^7}\\ +\frac{70z(z+c)^3}{R_2^9}\end{array}\right)\end{array}\right\}\\ \bar{\sigma}\left\{\begin{array}{l}(z-c)\left(\frac{-(1+2\tilde{\sigma})}{R_1^5}+\frac{5(z-c)^2}{R_1^7}\right)\\ +\frac{z'-((z-c)\hat{\sigma}+8z)}{R_2^5}\\ +5c(z+c)\left(\frac{-((3-4\tilde{\sigma})+3(3z+c))}{R_2^7}+\frac{14z(z+c)^2}{R_2^9}\right)\end{array}\right\}\end{array}\right] \tag{13}$$

$where \quad z'=\frac{z}{2}$

$$e_{zr}=e_{rz}$$
$$e_{\theta z}=0,\ e_{z\theta}=0$$
$$e_{\theta r}=0,\ e_{r\theta}=0 \tag{14}$$

*Case 2: when z=0*

Displacement components are given as

$$u_r=\frac{PU_0 ds\, r}{2\pi}\left[\frac{3c^2}{R^5}\left(\frac{\lambda}{\mu}+1\right)+\frac{2(1-\tilde{\sigma})}{R^3}\right] \tag{15}$$

$$u_z=\frac{PU_0 ds\, c}{4\pi}\left[\frac{\lambda}{\mu}\left(\frac{1}{R^3}-\frac{3c^2}{R^5}\right)\hat{\sigma}+2\left(\frac{(2\tilde{\sigma}-\bar{\sigma})}{R^3}+\frac{3\tilde{\sigma}\bar{\sigma}c^2}{R^5}\right)\right] \tag{16}$$

$where \ R=\sqrt{r^2+c^2}$

Strains components are given as

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$$e_{rr} = \frac{PU_0 ds}{8\pi}\left[12c^2\lambda\left(\frac{1}{R^5} - \frac{5r^2}{R^7}\right) + \mu\left(\begin{array}{l}\frac{8(1-\tilde{\sigma}) + 3\bar{\sigma}}{R^3} \\ + \frac{33c^2 - 3c\bar{\sigma}\left(\tilde{\sigma}c + r^2\left(8(1-\tilde{\sigma})^2 - 1\right)\right)}{R^5} \\ - \frac{60c^2r^2}{R^7}\end{array}\right)\right] \tag{17}$$

$$e_{zz} = \frac{PU_0 ds}{4\pi}\left[\lambda\bar{\sigma}c^2\left(\frac{(1+4\tilde{\sigma})}{R^5} - \frac{3c^2\tilde{\sigma}}{R^7}\right) + \mu\left(\begin{array}{l}\frac{4\tilde{\sigma}}{R^3} + \frac{(4(3-\tilde{\sigma}) + 2\tilde{\sigma}\bar{\sigma})}{R^5} \\ - \frac{30c^4\tilde{\sigma}\bar{\sigma}}{R^7}\end{array}\right)\right] \tag{18}$$

*NUMERICAL RESULTS*

We define dimensionless epicentral distance D, dimensionless radial displacement U, dimensionless vertical displacement i.e. uplift W and dimensionless radial strain E by the relations

$$D = \frac{r}{c},\; U = \frac{A}{c}u_r,\; W = \frac{A}{c}u_z,\; E = A\,e_{rr}$$

Where A is a dimensionless constant for each source, chosen in such a manner that W=1 at r= 0.

$$U = \frac{-(1-\tilde{\sigma})D}{((1-2\tilde{\sigma})(\lambda + (1-\tilde{\sigma})\mu) - 2\mu\tilde{\sigma})}\left[\frac{3(\lambda-\mu) + 2(1-\tilde{\sigma})(1+D^2)\mu}{(1+D^2)^{\frac{5}{2}}}\right] \tag{19}$$

$$W = \frac{-1}{2((1-2\tilde{\sigma})(\lambda + (1-\tilde{\sigma})\mu) - 2\mu\tilde{\sigma})}\left[\frac{\lambda(D^2-2)(1-2\tilde{\sigma}) + 2\mu\left((2\tilde{\sigma}(1-\tilde{\sigma}) - 1)(1+D^2) + 3\tilde{\sigma}\right)}{(1+D^2)^{\frac{5}{2}}}\right] \tag{20}$$

$$E = \frac{-1}{((1-2\tilde{\sigma})(\lambda + (1-\tilde{\sigma})\mu) - 2\mu\tilde{\sigma})}\left[\begin{array}{l}\frac{3(1-\tilde{\sigma})(1-4D^2)\lambda}{(1+D^2)^{\frac{7}{2}}} + \\ \mu\left(\frac{1}{(1+D^2)^{\frac{3}{2}}} + \frac{2(1-\tilde{\sigma})(5-\tilde{\sigma}) - \tilde{\sigma}}{(1+D^2)^{\frac{5}{2}}} - \frac{15(1-\tilde{\sigma})D^2}{(1+D^2)^{\frac{7}{2}}}\right)\end{array}\right] \tag{21}$$

$$A = \frac{-2\pi\mu c^3(1-\tilde{\sigma})}{PU_0 ds((1-2\tilde{\sigma})(\lambda + (1-\tilde{\sigma})\mu) - 2\mu\tilde{\sigma})} \tag{22}$$

## III. DISCUSSION AND CONCLUSION

Analytical expressions for the displacement and strain components due to five materials namely, Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone, Westerly Granite for drained behaviour for Tensile Dislocation in a poroelastic medium has been obtained.

For analysis taking P=1, c=1, z=1, $\frac{\lambda}{\mu} = \frac{2\tilde{\sigma}}{(1-2\tilde{\sigma})}$ and using Table 1

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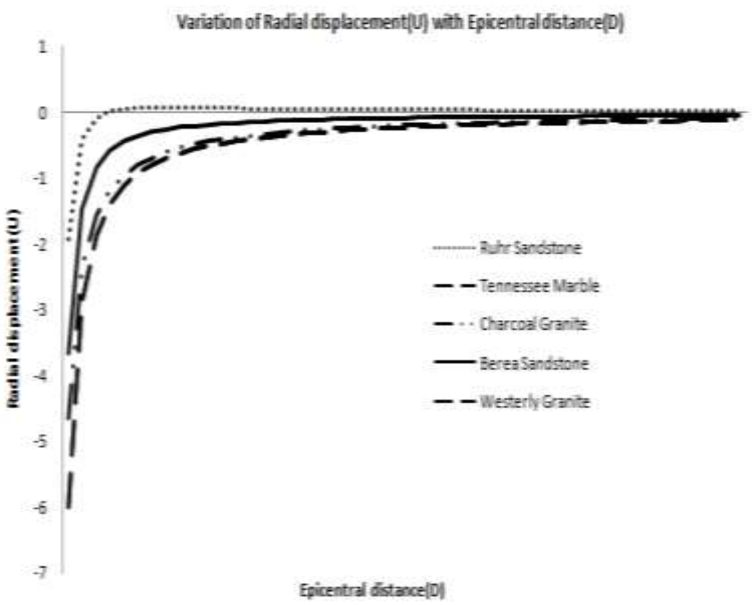
**Table:**

***Table 1. Material property***

| S No | Materials | Poisson Ratio $(\tilde{\sigma})$ |
|---|---|---|
| 1 | Ruhr Sandstone(RS) | 0.12 |
| 2 | Tennessee Marble(TM) | 0.25 |
| 3 | Charcoal Granite(CG) | 0.27 |
| 4 | Berea Sandstone(BS) | 0.20 |
| 5 | Westerly Granite(WG) | 0.25 |

Fig 4 shows the variation of dimensionless radial displacement with epicentral distance for drained behaviour of five materials i.e. RS, CG, TM, BS, WG. For all these materials we observe that as we move away from epicentre the displacement decrease gradually. The rate of decrease is less in case of RS as compared to TM & WG. Response of TM & CG is similar and shows more variation as compared to RS.
Fig 5 shows the variation of dimensionless vertical displacement (uplift) with epicentral distance. For all these materials, variation in vertical displacement (uplift) doesn't vary with the material i.e. response is quite similar, for TM, WG, BS materials. Rate of decrease is more in case of RS as compared to CG. Hence CG shows less variation as compared to RS.
Fig 6 shows the variation of dimensionless radial strain with epicentral distance for drained behaviour. For all these materials we observe that as we move away from epicentre the radial strain decrease gradually. The rate of decrease is more in case of RS as compared to CG.

**Figure 4:**




***Variation of radial displacement with epicentral distance***

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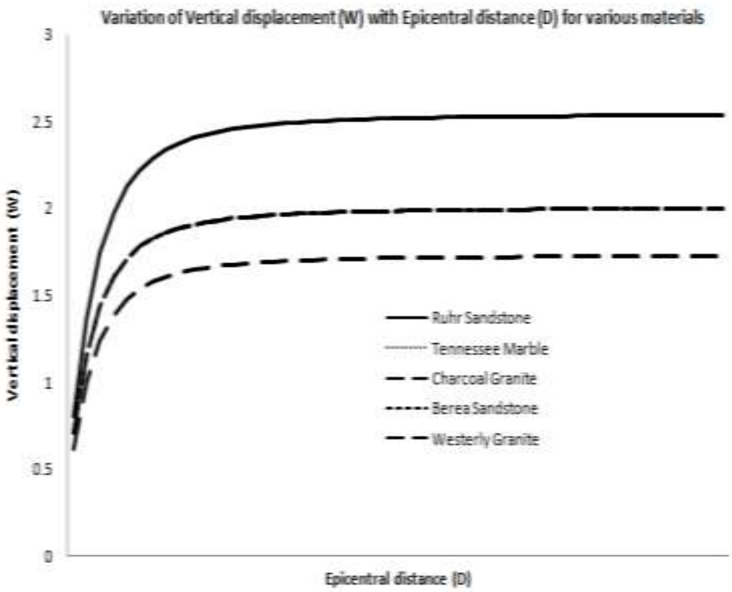
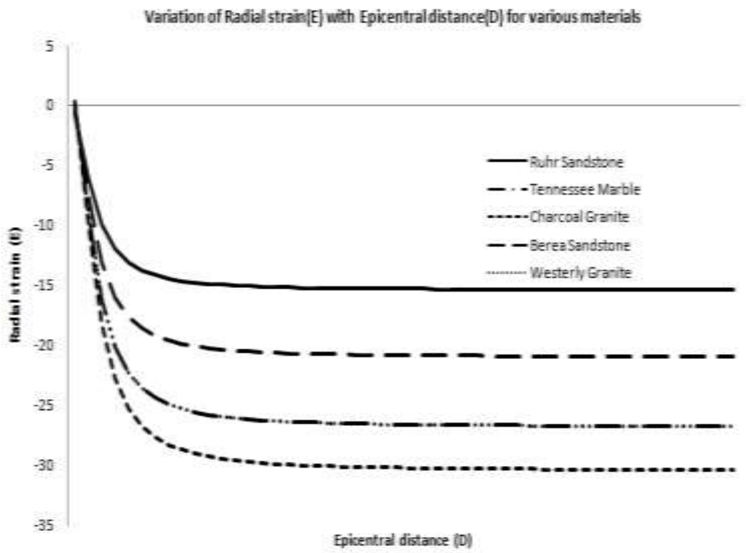
**Figure 5:**




*Variation of vertical displacement with epicentral distance*

**Figure 6:**




*Variation of radial strain with epicentral distance*

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